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Numerical Topology Optimization of Heat Sinks

IHTC-15 Conference (Kyoto, Japan)

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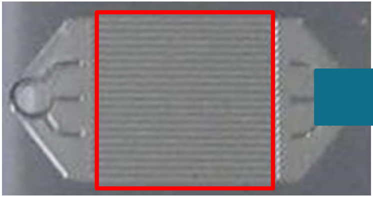
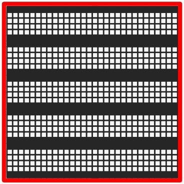
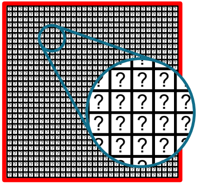
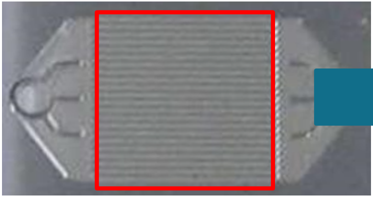
Topology optimization

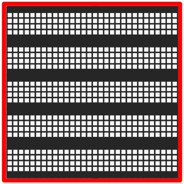




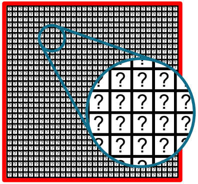


How to optimally distribute channels and fins?

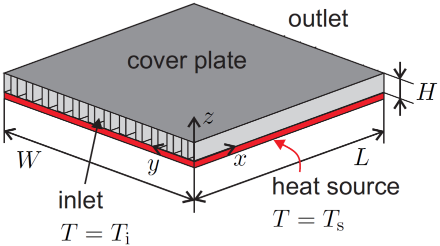




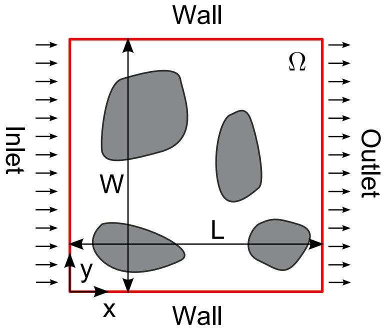
Aim: Generation of design concepts



How to optimally distribute channels and fins?



(a) 3D heat sink



(b) 2D heat sink

- Simulation of heat sink state $\phi(\mathbf{x})$ from porosity distribution $\varepsilon(\mathbf{x})$ based on two-dimensional model ($\phi = [\mathbf{v}, p, T]^\mathsf{T}$)
- Vertical profiles corresponding to fully developed flow through parallel plates

Field equations $\mathbf{M}(\varepsilon, \phi) = 0$:

- Hybrid: apply to *solid and fluid regions* by controlling parameters
- Averaged over height $\langle \cdot \rangle$: *2D simplification*

$$\nabla \cdot \langle \mathbf{v} \rangle = 0 \quad \text{(Continuity)}$$

$$\nabla \cdot \boxed{K_\mathrm{c}^\mathrm{m}} \rho \langle \mathbf{v} \rangle \langle \mathbf{v} \rangle - \nabla \cdot \mu \nabla \langle \mathbf{v} \rangle + \nabla \langle p \rangle + \boxed{\alpha} \langle \mathbf{v} \rangle = 0 \quad \text{(Momentum)}$$

$$\nabla \cdot \boxed{K_\mathrm{c}^\mathrm{e}} \rho c \langle T \rangle \langle \mathbf{v} \rangle - \nabla \cdot k \nabla \langle T \rangle + \underbrace{\boxed{k \frac{K_\mathrm{d}^\mathrm{e}}{H^2} \Big(\langle T \rangle - T_\mathrm{s} \Big)}}_{\text{source heat flux}} = 0 \quad \text{(Energy)}$$

Control parameters:

- Fluid: $\kappa = \infty$ & $k = k_\mathrm{f}$
- Solid: $\kappa = 0$ & $k = k_\mathrm{s}$

Averaging parameters:

- $K_\mathrm{c}^\mathrm{m} = 1.2$, $K_\mathrm{d}^\mathrm{m} = 12$, $K_\mathrm{c}^\mathrm{e} = 1.057$, $K_\mathrm{d}^\mathrm{e} = 2.569$
- $\alpha = \mu \left(\frac{1}{\kappa} + \frac{K_\mathrm{d}^\mathrm{m}}{H^2} \right)$

Porosity distribution $\varepsilon(\mathbf{x})$:

Solid material ($\varepsilon = 0$) $\longleftrightarrow$ Cooling fluid ($\varepsilon = 1$)

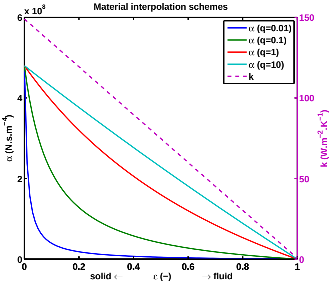
Mapping of physical parameters:

- Momentum loss coefficient α ('inverse permeability'):

$$\varepsilon \mapsto \alpha_q(\varepsilon) = \alpha_{\max} + (\alpha_{\min} - \alpha_{\max})\, \varepsilon \frac{1+q}{\varepsilon + q} \quad (*)$$

- Heat conductivity k:

$$\varepsilon \mapsto k(\varepsilon) = k_{\mathrm{s}} + (k_{\mathrm{f}} - k_{\mathrm{s}})\varepsilon$$



(*) Borrvall, Petersson (2003), "Topology optimization of fluids in Stokes flow"

For a general cost functional: $\mathcal{J}(\varepsilon, \phi) = \int_\Omega f \mathrm{d}\mathbf{x} + \int_{\partial\Omega} g \mathrm{d}\mathbf{s}$

Adjoint equations $\mathbf{M}_\phi^*(\varepsilon, \phi)\phi^* = \nabla_\phi \mathcal{J}$:

$$\boxed{\begin{aligned} -\nabla \cdot \mathbf{v}^* &= \frac{\partial f}{\partial p} \\ -K_\mathrm{c}^\mathrm{m} \rho \mathbf{v} \cdot (\nabla \mathbf{v}^* + (\nabla \mathbf{v}^*)^\intercal) - \nabla \cdot \mu \nabla \mathbf{v}^* - \nabla p^* + \alpha \mathbf{v}^* + K_\mathrm{c}^\mathrm{e} \rho c T^* \nabla T &= \frac{\partial f}{\partial \mathbf{v}} \\ -K_\mathrm{c}^\mathrm{e} \rho c \mathbf{v} \cdot \nabla T^* - \nabla \cdot k \nabla T^* + k \frac{K_\mathrm{d}^\mathrm{e}}{H^2} T^* &= \frac{\partial f}{\partial T} \end{aligned}}$$

Gradient of reduced cost functional $\hat{\mathcal{J}}(\varepsilon) = \mathcal{J}(\varepsilon, \phi(\varepsilon))$:

$$\nabla_\varepsilon \hat{\mathcal{J}}_\varepsilon = \boxed{\frac{\partial f}{\partial \varepsilon}} - \boxed{\frac{\partial \alpha}{\partial \varepsilon}} \mathbf{v} \cdot \mathbf{v}^* - \boxed{\frac{\partial k}{\partial \varepsilon}} \left(\nabla T \cdot \nabla T^* + \frac{K_\mathrm{d}^\mathrm{e}}{H^2} (T - T_\mathrm{s}) T^* \right)$$

Maximization of total heat transfer rate:

$$\boxed{\mathcal{J}(\varepsilon,\phi) = \int_A \dot{Q}''\mathrm{d}A = \int_\Omega k(\varepsilon)\frac{K_\mathrm{d}^\mathrm{e}}{H^2}\left(T_\mathrm{s}-T\right)\mathrm{d}\mathbf{x}}$$

Implementation:

- FVM solver on Cartesian, structured grid
- Forward model $\mathbf{M}(\varepsilon,\phi)=0$:
  1. Coupled flow system (laminar)
  2. Energy equation
- Adjoint model $\mathbf{M}_\phi^*(\varepsilon,\phi)\phi^* = \nabla_\phi\mathcal{J}$:
  1. Adjoint energy equation
  2. Coupled adjoint flow system
- Optimization with method of moving asymptotes (MMA):

  Svanberg (1987), "The method of moving asymptotes — A new method for structural optimization"

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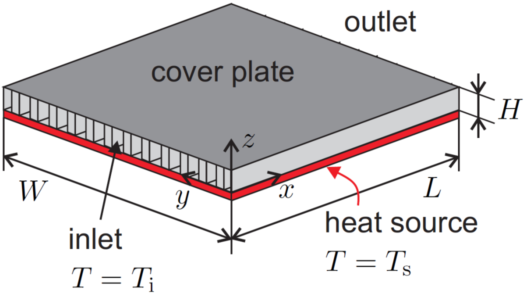


Figure: Maximizing total heat transfer rate of constant temperature heat source

Parameters

- Size: $L = W = 1\mathrm{cm}$, $H = 500\mu\mathrm{m}$
- Materials: Si, H_2O
- Pressure head: $\Delta p = 10\mathrm{kPa}$
- Temperatures: $T_\mathrm{i} = 0\mathrm{K}$, $T_\mathrm{s} = 40\mathrm{K}$
- Stokes flow $(\mathrm{Re} = 0)$
- $\alpha_\mathrm{min} = \mu \frac{K_\mathrm{d}^\mathrm{m}}{H^2}$, $\alpha_\mathrm{max} = \frac{K_\mathrm{d}^\mathrm{m}}{H^2}$
- $q_\mathrm{init} = 0.01$, $q_\mathrm{final} = 1$, $N_q = 50$, $N = 100$
- Grid size: 200×200

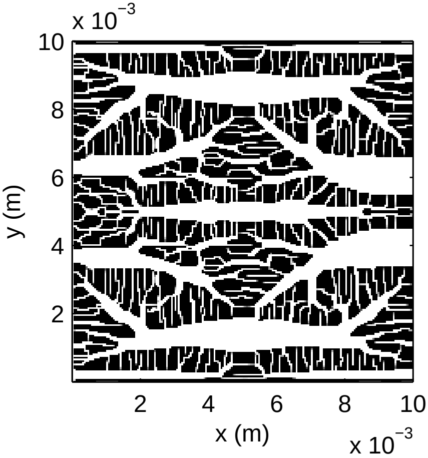


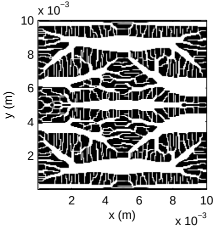
Figure: Porosity after 100 iterations

Realistic

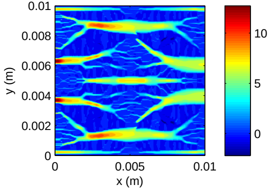
- Total heat transfer rate: $\dot{Q} = 794W$
- Several scales of channel diameters

R_{hs}	0.0504	K/W
$\dot{m}$	8.88	g/s
$\dot{m}c$	37.1	W/K
hA	28.4	W/K

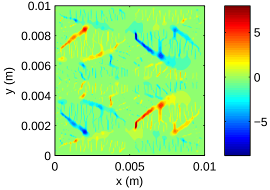
Flow and heat transfer in the optimized heat sink



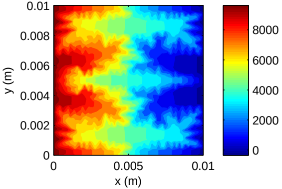
(a) Optimized design



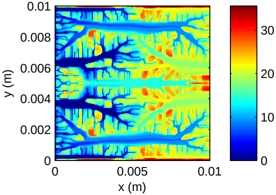
(b) Velocity (→) (m/s)



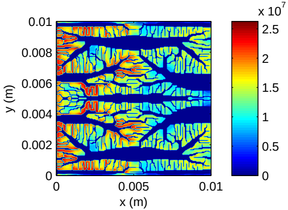
(c) Velocity (↑) (m/s)



(d) Pressure (Pa)



(e) Temperature (K)



(f) Source heat flux (W/m^2)

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Conclusions

- Two-dimensional heat sink model for constant temperature heat source
- Pixel-based representation of geometries gives flexible parameterization, very suitable for topology optimization
- Continuous adjoint method efficiently calculates sensitivity
- Application on practical micro heat sink shows that topology optimization is very promising and powerful design method

Further work

- Tackle grid dependence: grid refinement generates finer heat sink structure
- Resolve dependency on α_{solid} and q

- Van Oevelen T., Baelmans M. (2014), "Numerical topology optimization of heat sink" *IHTC-15 Conference, Kyoto, Japan*
- Van Oevelen T., Baelmans M. (2014), "Application of topology optimization in a conjugate heat transfer problem" *OPT-i Conference, Kos, Greece*

THANK YOU!

Adjoint method applied to linearized model $\mathbf{M}_\phi(\varepsilon,\phi)\phi' + \mathbf{M}_\varepsilon(\varepsilon,\phi)\varepsilon' = 0$:

$$\boxed{\int_\Omega \phi^* \cdot \mathbf{M}_\phi(\varepsilon,\phi)\phi' \mathrm{d}\mathbf{x} = \int_\Omega \phi' \cdot \mathbf{M}_\phi^*(\varepsilon,\phi)\phi^* \mathrm{d}\mathbf{x}}$$

Derivative of reduced objective functional: $\hat{\mathcal{J}}(\varepsilon) = \mathcal{J}(\varepsilon, \phi(\varepsilon))$:

$$\frac{\mathrm{d}\hat{\mathcal{J}}}{\mathrm{d}\varepsilon}\varepsilon' = \int_\Omega \left(\nabla_\varepsilon \mathcal{J}\varepsilon' + \nabla_\phi \mathcal{J}\phi'\right) \mathrm{d}\mathbf{x} \qquad = \boxed{\int_\Omega \left(\nabla_\varepsilon \mathcal{J} + \nabla_\phi \mathcal{J}\frac{\partial \phi}{\partial \varepsilon}\right)\varepsilon' \mathrm{d}\mathbf{x}}$$

$$= \int_\Omega \left(\nabla_\varepsilon \mathcal{J}\varepsilon' + \phi^* \mathbf{M}_\phi \phi'\right) \mathrm{d}\mathbf{x} \qquad (\mathbf{M}_\phi^* \phi^* = \nabla_\phi \mathcal{J})$$

$$= \boxed{\int_\Omega \underbrace{\left(\nabla_\varepsilon \mathcal{J} - \mathbf{M}_\varepsilon^* \phi^*\right)}_{\nabla_\varepsilon \hat{\mathcal{J}}} \varepsilon' \mathrm{d}\mathbf{x}} \qquad (\mathbf{M}_\phi \phi' = -\mathbf{M}_\varepsilon \varepsilon')$$

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$$\alpha_q(\varepsilon) = \alpha_{\max} + (\alpha_{\min} - \alpha_{\max})\, \varepsilon \frac{1+q}{\varepsilon + q}$$

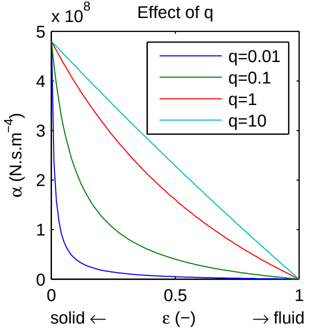
Updating procedure

Set q_{init}, q_{final} $(> q_{\text{init}})$ and N_q:

$$q_1 = q_{\text{init}}, \qquad \beta = \left(\frac{q_{\text{final}}}{q_{\text{init}}}\right)^{\frac{1}{N_q - 1}}$$

For $k = 1, \ldots, N_{\text{tot}}$, do:

$$q_{k+1} = \min(\beta q_k, q_{\text{final}})$$



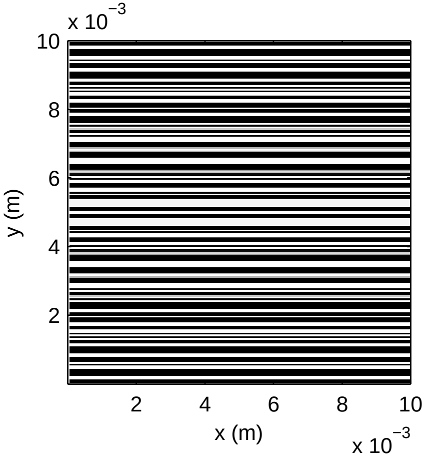


Figure: Porosity after 100 iterations

Parallel microchannels

- Total heat transfer rate: $\dot{Q} = 347W$ ($\dot{Q} = 794W$)

R_{hs}	0.1152	0.0504	K/W
$\dot{m}$	3.91	8.88	g/s
$\dot{m}c$	16.3	37.1	W/K
hA	12.4	28.4	W/K

Unrestricted design

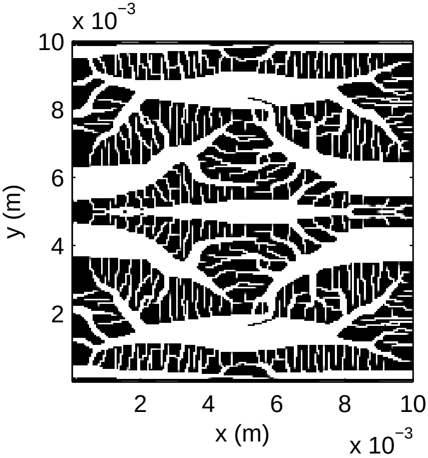


Figure: Porosity after 100 iterations

Unrealistic inlet conduction

- Total heat transfer rate: $\dot{Q} = 836W$
  - Advection outlet: $794W$
  - Conduction inlet: $41.4W$
- Cause: Inlet T b.c.
- Solution: Collector model
- *Quick fix: set $\varepsilon = 1$ in first cell*